Polynomekvationer

fall ①

$x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$

fall ②

$x^2 - 5x = 0$

$x(x-5) = 0$

$x_1 = 0 \quad x_2 = 5$

fall ③

$x^2 - 4x - 5 = 0$

$x = 2 \pm \sqrt{2^2 + 5}$

$= 2 \pm \sqrt{9}$

$= 2 \pm 3$

$x_1 = 5 \quad x_2 = -1$

$x^2 + px + q = 0$

$x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$

fall ④

$(x-1)(x+3) = 0$

När är parenteserna lika med noll?

$x_1 = 1 \quad x_2 = -3$

fall ⑤

$x^3 - 2x^2 + x = 0$

Faktorisera uttryckket

$x(x^2 - 2x + 1) = 0 \qquad x_1 = 0$

$x^2 - 2x + 1 = 0$

$x = 1 \pm \sqrt{1 - 1}$

$= 1 \pm \sqrt{0} \qquad x_2 = 1$

Svar: $x_1 = 0$, $x_2 = 1$

fall ⑥ $(x-1)(x+1)(x-3)(x+3) = 0$

$x_1 = 1, \; x_2 = -1, \; x_3 = 3, \; x_4 = -3$

fall (7) Ekvationslösning med substitution

$x^4 - 5x^2 + 4 = 0$ Vi gör substitutionen $t = x^2$

$(x^2)^2 - 5x^2 + 4 = 0$

$t^2 - 5t + 4 = 0$

$t = \frac{5}{2} \pm \sqrt{\left(\frac{5}{2}\right)^2 - 4}$

$= \frac{5}{2} \pm \sqrt{\frac{25}{4} - 4}$

$= \frac{5}{2} \pm \sqrt{\frac{25}{4} - \frac{16}{4}}$

$= \frac{5}{2} \pm \sqrt{\frac{9}{4}}$

$= \frac{5}{2} \pm \frac{3}{2}$

$t_1 = \frac{8}{2} = 4$

$t_2 = \frac{2}{2} = 1$

$t_1 = 1 \Rightarrow 1 = x^2$ $x_1 = 1$, $x_2 = -1$

$t_2 = 4 \Rightarrow 4 = x^2$ $x_3 = 2$, $x_4 = -2$

Annat exempel

$\left(\sqrt{x+1}\right)^2 - 10\sqrt{x+1} + 9 = 0$ Substitution: $\sqrt{x+1} = t$

$t^2 - 10t + 9 = 0$

$t = 5 \pm \sqrt{5^2 - 9}$

$= 5 \pm \sqrt{16}$

$= 5 \pm 4$

$t_1 = 9$

$t_2 = 1$

$t = 9 \Rightarrow \sqrt{x+1} = 9$

$x + 1 = 81$

$x = 80$ Undersök falskrot

$t = 1 \Rightarrow \sqrt{x+1} = 1$

$x + 1 = 1$

$x = 0$ Undersök falskrot

Svar: $x_1 = 80$, $x_2 = 0$