Exempelprov

1. a) $f(x) = 2x^3 + 4x^2 - 150$

$f'(x) = 6x^2 + 8x$ 1E

b) $f(x) = \frac{1}{3} \cdot x^3 + 100x^2 - 12x + 1$

$f'(x) = x^2 + 200x - 12$ 1E

c) $f(x) = 30e^{2x} + e^{x} + e^{-x}$

$f'(x) = 60e^{2x} + e^{x} - e^{-x}$ 1E

d) $f(x) = e^{10x} + 4e^{10x} + 22x - 11$

$f'(x) = 10e^{10x} + 40e^{10x} + 22$ 1E

e) $f(x) = 10\sqrt{x} + \frac{1}{x^4} - \frac{3}{x}$

$= 10x^{\frac{1}{2}} + x^{-4} - 3x^{-1}$

$f'(x) = 5x^{-\frac{1}{2}} - 4x^{-5} + 3x^{-2}$

$= \frac{1}{5\sqrt{x}} - \frac{4}{x^5} + \frac{3}{x^2}$ 1C

f) $f(x) = \frac{4}{15} x^{\frac{21}{3}} + \frac{10}{7} \cdot x^{\frac{1}{7}}$

$f'(x) = \frac{84}{45} x^{\frac{18}{3}} + \frac{10}{49} x^{\frac{-6}{7}}$

$= \frac{84}{45} x^{6} + \frac{10}{49} x^{\frac{-6}{7}}$ 1C

2. a) $\frac{x^2 + 5}{x^2 - 16}$

odef. då $x^2 - 16 = 0$

$x^2 - 16 = 0 \quad x = \pm 4$ Svar: $x = \pm 4$ 1E

b) $\frac{x - 1}{x^3 - 4x^2 - 12x}$

odef. då $x^3 - 4x^2 - 12x = 0$

$x^3 - 4x^2 - 12x = 0$

$x(x^2 - 4x - 12) = 0 \qquad x_1 = 0$ 1E

$x^2 - 4x - 12 = 0$

$x = 2 \pm \sqrt{\left(\frac{4}{2}\right)^2 + 12} \qquad x_2 = 6$

$= 2 \pm \sqrt{16} \qquad x_3 = -2$

$= 2 \pm 4$

Svar: $\begin{cases} x_1 = 0 \\ x_2 = 6 \\ x_3 = -2 \end{cases}$ 1C

3. a) $|x| = 7$

$x = \pm 7$

Svar: $x = \pm 7$ 1E

b) $|x+3| = 8$

två fall:

I $x + 3 = 8$ $x = 5$

II $x + 3 = -8$ $x = -11$

Svar: $\begin{cases} x_1 = 5 \\ x_2 = -11 \end{cases}$ 1C

c) $|x^2 - 8| = 1$

två fall:

I $x^2 - 8 = 1$ $x^2 = 9$ $x = \pm 3$

1C II $x^2 - 8 = -1$ $x^2 = 7$ $x = \pm\sqrt{7}$

Svar: $\begin{cases} x_{1,2} = \pm 3 \\ x_{3,4} = \pm\sqrt{7} \end{cases}$ 1A

4. $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

a) $f'(1) = 2 \cdot 1 - 4 = -2$ $f'(1) < 0$ 1E

b) $f'(-2) = 2 \cdot (-2) - 4 = -8$ $f'(-2) < 0$ 1E

c) $f'(2) = 2 \cdot 2 - 4 = 0$ $f'(2) = 0$ 1E

d) $f'(\frac{1}{2}) = 2 \cdot \frac{1}{2} - 4 = -3$ $f'(\frac{1}{2}) < 0$ 1E

1c

5. T.ex $f(x) = x^2 + x$ finns oändligt många

$f'(x) = 2x + 1$ $f'(1) = 2 \cdot 1 + 1 = 3$ 1c Motivering

6. vinkelräta: $k_1 \cdot k_2 = -1$ $\quad k = \dfrac{y_2 - y_1}{x_2 - x_1}$

$L_1: k_1 = \dfrac{12-4}{3-1} = \dfrac{8}{2} = 4$ 1E

$L_2: k_2 = \dfrac{\frac{-3}{12} + \frac{1}{12}}{1+1} = \dfrac{\left(\frac{-2}{12}\right)}{\frac{2}{1}} = \dfrac{-1}{12}$ 1E

$k_1 \cdot k_2 \neq -1$ de är inte vinkelräta 1E

7. $f(x) = x^2 + 2x - 3$ $\quad$ Viktiga punkter: Nollställen, skärning i y, vertex

$f(x) = 0 \quad x^2 + 2x - 3 = 0$

$x = -1 \pm \sqrt{1+3}$

$= -1 \pm 2$

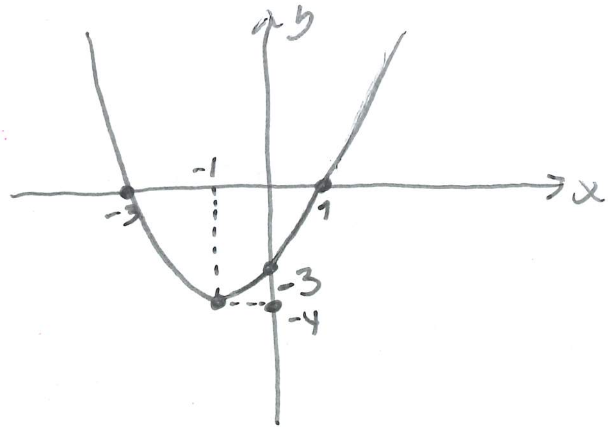
$x_1 = 1$

$x_2 = -3$ 1E

Vertex: symmetrilinjen: $x = -1$

$f(-1) = 1 + 2 \cdot (-1) - 3 = -4$ 1E

skär y i $y = -3$



1E, 1C

8. a)
1E $\quad$ 1C

$\dfrac{x^3 - 4x}{x^3 - 2x^2 - 8x} = \dfrac{\cancel{x}(x^2-4)}{\cancel{x}(x^2-2x-8)} = \dfrac{(x-2)(x+2)}{x^2-2x-8} = \dfrac{(x-2)\cancel{(x+2)}}{(x-4)\cancel{(x+2)}} = \underline{\dfrac{(x-2)}{(x-4)}}$

1C

Skriv $x^2 - 2x - 8$ i faktorform:

$x^2 - 2x - 8 = 0 \quad x = 1 \pm \sqrt{1+8}$

$= 1 \pm 3 \quad x_1 = 4$

$x_2 = -2$

$x^2 - 2x - 8 = (x-4)(x+2)$

8\. b) $$\frac{x^4-20x^2+64}{x^2+2x-8}$$

Skriv täljare och nämnare i faktorform

$$x^4-20x^2+64=0 \qquad x^2=t \qquad t^2-20t+64=0$$

$$t=10\pm\sqrt{100-64}$$

$$=10\pm\sqrt{36}$$

$$=10\pm 6 \qquad t_1=4 \qquad t_2=16$$

$$x_{1,2}^2=4 \qquad x_{1,2}=\pm 2$$

$$x_{3,4}^2=16 \qquad x_{3,4}=\pm 4$$

$$x^4-20x^2+64=(x-2)(x+2)(x-4)(x+4)$$ 2c

$$x^2+2x-8=0$$

$$x=-1\pm\sqrt{1+8}$$

$$=-1\pm 3 \qquad x_1=4 \qquad x_2=2$$

$$x^2+2x-8=(x+4)(x-2)$$ 1c

$$\frac{x^4-20x^2+64}{x^2+2x-8}=\frac{\cancel{(x-2)}(x+2)(x-4)\cancel{(x+4)}}{\cancel{(x+4)}\cancel{(x-2)}}=x^2-2x-8$$

1c

9\. a) $$(x-2)(x+3)(x^2-16)=0$$

$$x-2=0 \qquad x_1=2$$

$$x+3=0 \qquad x_2=-3$$

$$x^2-16=0 \qquad x_{3,4}=\pm 4$$ 1c

9. b) $\sqrt{-x^2+20} = x+2$

$-x^2+20 = x^2+4x+4$

$2x^2+4x-16=0$

$x^2+2x-8=0$

$x = -1 \pm \sqrt{1+8}$

$= -1 \pm 3$

$x_1 = 2$ 1E Undersök falskrot!

$x_2 = -4$

$x=2$ $\sqrt{-4+20} = 2+2$

$\sqrt{16} = 4$

stämmer!

$x=-4$ $\sqrt{-16+20} = -4+2$

$\sqrt{4} = -2$

stämmer inte $x=-4$ falsk rot!

1C

10. $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$f(x)=x^2$ $x=2$ $f'(2) = \lim_{h \to 0} \frac{(2+h)^2-2^2}{h}$ 1E

$= \lim_{h \to 0} \frac{2^2+4h+h^2-2^2}{h} = \lim_{h \to 0} \frac{4h+h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(4+h)}{\cancel{h}} = 4$

Svar: $f'(2)=4$ 1E, 1C

11. a) $x^2+6x-16$

$x^2+6x-16=0$

$x=-3\pm\sqrt{9+16}$

$=-3\pm\sqrt{25}$

$=-3\pm5$ $\quad x_1=-8$

$x_2=2$

$x^2+6x-16=(x-2)(x+8)$ 1E

b) x^3-8x^2-9x

$x^3-8x^2-9x=0$

$x(x^2-8x-9)=0$ $\quad x_1=0$

$x^2-8x-9=0$

$x=4\pm\sqrt{16+9}$

$x=4\pm5$ $\quad x_2=9$

$x_3=-1$

$x^3-8x^2-9x=x(x-9)(x+1)$ 1C

c) $2x^4+20x^3+18x^2$

$2x^4+20x^3+18x^2=0$

$2x^2(x^2+10x+18)$ $\quad x_1^2=0$

$x^2+10x+18$

$x=-5\pm\sqrt{25-18}$ $\quad x_2=-5+\sqrt{7}$

$x_3=-5-\sqrt{7}$

$2x^4+20x^3+18x^2=$

$=x^2(x+5-\sqrt{7})(x+5+\sqrt{7})$

2C

12. $f(x) = \sqrt{x} + 10x = x^{\frac{1}{2}} + 10x$

$f'(x) = \frac{1}{2} \cdot x^{-\frac{1}{2}} + 10 = \frac{1}{2\sqrt{x}}$ 1E $f'(4) = \frac{1}{2\sqrt{4}} + 10 = \frac{1}{4} + 10$

$= \frac{41}{4}$ ← k-värdet!

Sök y-värdet för $y = kx + m$

$f(4) = \sqrt{4} + 40 = 42$ 1C

$y = kx + m \Rightarrow 42 = \frac{41}{4} \cdot 4 + m$

$42 = 41 + m$

$m = 1$

$\underline{y = \frac{41}{4}x + 1}$ 1C

13. $\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$ $f(x) = ax^2 + bx + c$

$f'(x) = \lim_{h \to 0} \frac{a(x+h)^2 + b(x+h) + c - (ax^2 + bx + c)}{h}$ 1C

$= \lim_{h \to 0} \frac{\cancel{ax^2} + 2axh + ah^2 + \cancel{bx} + bh + \cancel{c} - \cancel{ax^2} - \cancel{bx} - \cancel{c}}{h}$

$= \lim_{h \to 0} \frac{2axh + ah^2 + bh}{h} = \lim_{h \to 0} \frac{\cancel{h}(2ax + ah + b)}{\cancel{h}} = \lim_{h \to 0} 2ax + ah + b$

$= \underline{2ax + b}$ 1A 1A Snygg och korrekt kommunicerat

14. a) $\lim_{h\to -2} \frac{h^4-2h^3-8h^2}{h^4-4h^2}$ Nämnaren går mot noll då $h \to -2$

$\lim_{h\to -2} \frac{h^2(h^2-2h-8)}{h^2(h^2-4)} = \lim_{h\to -2} \frac{h^2-2h-8}{h^2-4} = \lim_{h\to -2} \frac{h^2-2h-8}{(h-2)(h+2)}$ 1C

Skriv h^2-2h-8 i faktorform

$h^2-2h-8=0$

$h = 1 \pm \sqrt{1+8}$
$= 1 \pm 3$

$h_1 = 4$
$h_2 = -2$

$\lim_{h\to -2} \frac{(h-4)\cancel{(h+2)}}{(h-2)\cancel{(h+2)}} = \lim_{h\to -2} \frac{(h-4)}{(h-2)} = \frac{-6}{-4}$

$= \frac{3}{2}$ 2C

b) $\lim_{x\to\infty} \frac{x^3+x^2+x+1}{35x^3-2x^2+4x-2} = \lim_{x\to\infty} \frac{x^3(1+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3})}{x^3(35-\frac{2}{x}+\frac{4}{x^2}-\frac{2}{x^3})}$

$= \lim_{x\to\infty} \frac{1+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}}{35-\frac{2}{x}+\frac{4}{x^2}-\frac{2}{x^3}}$ 1A Då $x\to\infty$ försummas alla termer med x i nämnaren

$= \frac{1}{35}$ 1A

c) $\lim_{x\to\infty} \frac{\sqrt{16x^4-43x^3}}{\sqrt{25x^4+1643x^3}} = \lim_{x\to\infty} \sqrt{\frac{16x^4-43x^3}{25x^4+1643x^3}} = \lim_{x\to\infty} \sqrt{\frac{x^4(16-\frac{43}{x})}{x^4(25+\frac{1643}{x})}}$

1A

$= \lim_{x\to\infty} \sqrt{\frac{16-\frac{43}{x}}{25-\frac{1643}{x}}}$ Då $x\to\infty$ försummas $\frac{43}{x}$ och $\frac{1643}{x}$

$= \sqrt{\frac{16}{25}} = \frac{4}{5}$ 1A

15. $f(x) = k(x+a)(x+2a) = k(x^2+3ax+2a^2)$ 1C

$f(a) = k(a^2+3a^2+2a^2) = 6ka^2 = 18$

$f'(x) = 2kx + 3ka$

$f'(a) = 2ka + 3ka = 5ka = 15$

$\begin{cases} 6ka^2 = 18 \\ 5ka = 15 \end{cases}$

1A

$k = \frac{15}{5a} = \frac{3}{a}$

$\frac{3}{a} \cdot 6a^2 = \frac{18a^2}{a} = 18$

$18a = 18 \quad a = 1$

$5k = 15 \quad k = 3$

$f(x) = 3(x+1)(x+2) = 3(x^2+3x+2) = \underline{3x^2+9x+6}$ 1A

16. $f(x) = k(x-b)(x-5b) = k(x^2 - 6bx + 5b^2)$

$= kx^2 - 6kbx + 5kb^2$ 1C

$f'(x) = 2kx - 6kb$

$T_1: f'(1) = 2k - 6kb = 4$

$T_2: f'\left(\frac{25}{8}\right) = \frac{25k}{4} - 6kb = -\frac{1}{4}$

om T_1 och T_2 ska vara vinkelräta krävs $k_1 \cdot k_2 = -1$

$4 \cdot k_2 = -1 \quad k_2 = -\frac{1}{4}$ 1A

$f'(1) - f'\left(\frac{25}{8}\right) = 2k - 6\cancel{k}b - \left(\frac{25k}{4} - 6\cancel{k}b\right) = 2k - \frac{25k}{4} = 4 - \left(-\frac{1}{4}\right)$

$2k - \frac{25k}{4} = \frac{16}{4} + \frac{1}{4} = \frac{17}{4}$

$k\left(2 - \frac{25}{4}\right) = \frac{17}{4}$

$k\left(\frac{-17}{4}\right) = \frac{17}{4} \qquad k = \frac{\frac{17}{4}}{\frac{-17}{4}} = -1$ 1A

$-2 + 6b = 4 \qquad b = 1$

$f(x) = -x^2 + 6x - 5$ 1A